Access to Science, Engineering and Agriculture: Mathematics 1 MATH00030 Chapter 7 Solutions

In all these solutions, c will represent an arbitrary constant.

1. (a) Since $f(x) = 5$ is a constant, $\displaystyle\int_0^1 5\,dx = [5x]_0^1 = 5$.

   (b) Since $f(x) = -\pi\cos(e)$ is a constant, $\displaystyle\int -\pi\cos(e)\,dx = -\pi\cos(e)x + c$.

   (c) Since $f(x) = x^2$ is of the form $f(x) = x^n$ with $n = 2$,

   $$\int_{-1}^{1} x^2\,dx = \left[\frac{x^{2+1}}{2+1}\right]_{-1}^{1} = \left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{2}{3}.$$

   (d) Since $f(x) = x^{\frac{9}{2}}$ is of the form $f(x) = x^n$ with $n = \dfrac{9}{2}$,

   $$\int x^{\frac{9}{2}}\,dx = \frac{1}{9/2+1}x^{\frac{9}{2}+1} + c = \frac{2}{11}x^{\frac{11}{2}} + c.$$

   (e) Since $f(x) = x^{-5}$ is of the form $f(x) = x^n$ with $n = -5$,

   $$\begin{aligned}\int_1^2 x^{-5}\,dx &= \left[\frac{1}{-5+1}x^{-5+1}\right]_1^2\\ &= \left[-\frac{1}{4}x^{-4}\right]_1^2\\ &= -\frac{1}{4}2^{-4} - \left(-\frac{1}{4}1^{-4}\right)\\ &= -\frac{1}{64} + \frac{1}{4}\\ &= \frac{15}{64}.\end{aligned}$$

   (f) Since $f(x) = x^{\cos(2)}$ is of the form $f(x) = x^n$ with $n = \cos(2)$,

   $$\int x^{\cos(2)}\,dx = \frac{1}{\cos(2)+1}x^{\cos(2)+1} + c.$$

   (g) Since $f(x) = e^{4x}$ is of the form $f(x) = e^{ax}$ with $a = 4$,

   $$\int_0^2 e^{4x}\,dx = \left[\frac{1}{4}e^{4x}\right]_0^2 = \frac{1}{4}e^{4(2)} - \frac{1}{4}e^0 = \frac{1}{4}(e^8 - 1).$$

(h) Since $f(x) = e^{\frac{3}{2}x}$ is of the form $f(x) = e^{ax}$ with $a = \frac{3}{2}$,

$$\int e^{\frac{3}{2}x}\,dx = \frac{1}{3/2}e^{\frac{3}{2}x} + c = \frac{2}{3}e^{\frac{3}{2}x} + c.$$

(i) Since $f(x) = e^{-6x}$ is of the form $f(x) = e^{ax}$ with $a = -6$,

$$\begin{aligned}\int_{-1}^{0} e^{-6x}\,dx &= \left[\frac{1}{-6}e^{-6x}\right]_{-1}^{0}\\ &= \left[-\frac{1}{6}e^{-6x}\right]_{-1}^{0}\\ &= -\frac{1}{6}e^{0} - \left(-\frac{1}{6}e^{-6(-1)}\right)\\ &= \frac{1}{6}\left(e^{6} - 1\right).\end{aligned}$$

(j) Since $f(x) = e^{\pi x}$ is of the form $f(x) = e^{ax}$ with $a = \pi$, $\int e^{\pi x}\,dx = \frac{1}{\pi}e^{\pi x} + c$.

(k) $\int_{1}^{2} \frac{1}{x}\,dx = [\ln(x)]_{1}^{2} = \ln(2) - \ln(1) = \ln(2) - 0 = \ln(2)$.

(l) Since $f(x) = \sin(2x)$ is of the form $f(x) = \sin(ax)$ with $a = 2$,

$$\int \sin(2x)\,dx = -\frac{1}{2}\cos(2x) + c.$$

(m) Since $f(x) = \sin(-3x)$ is of the form $f(x) = \sin(ax)$ with $a = -3$,

$$\begin{aligned}\int_{0}^{\frac{\pi}{3}} \sin(-3x)\,dx &= \left[-\frac{1}{-3}\cos(-3x)\right]_{0}^{\frac{\pi}{3}}\\ &= \left[\frac{1}{3}\cos(-3x)\right]_{0}^{\frac{\pi}{3}}\\ &= \frac{1}{3}\cos\left(-3\cdot\frac{\pi}{3}\right) - \frac{1}{3}\cos(-3(0))\\ &= \frac{1}{3}\cos\left(-\pi\right) - \frac{1}{3}\cos(0)\\ &= \frac{1}{3}(-1) - \frac{1}{3}(1)\\ &= -\frac{2}{3}.\end{aligned}$$

(n) Since $f(x) = \sin(ex)$ is of the form $f(x) = \sin(ax)$ with $a = e$,

$$\int \sin(ex)\,dx = -\frac{1}{e}\cos(ex) + c.$$

(o) Since $f(x) = \cos(3x)$ is of the form $f(x) = \cos(ax)$ with $a = 3$,

$$\begin{aligned}\int_{-\pi}^{\frac{\pi}{3}} \cos(3x)\,dx &= \left[\frac{1}{3}\sin(3x)\right]_{-\pi}^{\frac{\pi}{3}} \\ &= \frac{1}{3}\sin\left(3\cdot\frac{\pi}{3}\right) - \frac{1}{3}\sin(3(-\pi)) \\ &= \frac{1}{3}\sin(\pi) - \frac{1}{3}\sin(-3\pi) \\ &= \frac{1}{3}(0) - \frac{1}{3}(0) \\ &= 0.\end{aligned}$$

(p) Since $f(x) = \cos(-\pi x)$ is of the form $f(x) = \cos(ax)$ with $a = -\pi$,

$$\int \cos(-\pi x)\,dx = \frac{1}{-\pi}\sin(-\pi x) + c = -\frac{1}{\pi}\sin(-\pi x) + c.$$

2. (a) We will first use the sum and multiple rules to find the corresponding definite integral.

$$\begin{aligned}&\int 1 + 3x - 2x^2 + 3x^3 - 4x^4\,dx \\ &\qquad = \int 1\,dx + \int 3x\,dx + \int -2x^2\,dx + \int 3x^3\,dx + \int -4x^4\,dx \\ &\qquad = \int 1\,dx + 3\int x\,dx - 2\int x^2\,dx + 3\int x^3\,dx - 4\int x^4\,dx \\ &\qquad = x + 3\left(\frac{1}{2}x^2\right) - 2\left(\frac{1}{3}x^3\right) + 3\left(\frac{1}{4}x^4\right) - 4\left(\frac{1}{5}x^5\right) + c \\ &\qquad = x + \frac{3}{2}x^2 - \frac{2}{3}x^3 + \frac{3}{4}x^4 - \frac{4}{5}x^5 + c.\end{aligned}$$

Note that in your assignment or exam solutions you don't need to give as much detail as this. I am just setting out everything carefully until you get used to the ideas involved.
Hence

$$\begin{aligned}\int_{-1}^{1} 1 + 3x - 2x^2 + 3x^3 - 4x^4\,dx &= \left[x + \frac{3}{2}x^2 - \frac{2}{3}x^3 + \frac{3}{4}x^4 - \frac{4}{5}x^5\right]_{-1}^{1} \\ &= 1 + \frac{3}{2}(1^2) - \frac{2}{3}(1^3) + \frac{3}{4}(1^4) - \frac{4}{5}(1^5) \\ &\quad - \left[-1 + \frac{3}{2}(-1)^2 - \frac{2}{3}(-1)^3 + \frac{3}{4}(-1)^4 - \frac{4}{5}(-1)^5\right] \\ &= \frac{107}{60} - \frac{163}{60} \\ &= -\frac{14}{15}.\end{aligned}$$

(b) Using the sum and multiple rules,

$$\begin{aligned}\int -x^{-1} + 2\sin 4x \, dx &= \int -x^{-1} \, dx + \int 2\sin 4x \, dx \\ &= -\int x^{-1} \, dx + 2\int \sin 4x \, dx \\ &= -\int \frac{1}{x} \, dx + 2\int \sin 4x \, dx \\ &= -\ln(x) + 2\left(-\frac{1}{4}\cos(4x)\right) + c \\ &= -\ln(x) - \frac{1}{2}\cos(4x) + c.\end{aligned}$$

(c) We will first use the sum and multiple rules to find the corresponding definite integral.

$$\begin{aligned}\int 3e^{-\frac{1}{2}x} - 2\cos\left(\frac{1}{2}x\right) \, dx &= \int 3e^{-\frac{1}{2}x} \, dx + \int -2\cos\left(\frac{1}{2}x\right) \, dx \\ &= 3\int e^{-\frac{1}{2}x} \, dx - 2\int \cos\left(\frac{1}{2}x\right) \, dx \\ &= 3\left(\frac{1}{-1/2}e^{-\frac{1}{2}x}\right) - 2\left(\frac{1}{1/2}\sin\left(\frac{1}{2}x\right)\right) + c \\ &= -6e^{-\frac{1}{2}x} - 4\sin\left(\frac{1}{2}x\right).\end{aligned}$$

Hence

$$\begin{aligned}\int_0^{\pi} 3e^{-\frac{1}{2}x} - 2\cos\left(\frac{1}{2}x\right) \, dx &= \left[-6e^{-\frac{1}{2}x} - 4\sin\left(\frac{1}{2}x\right)\right]_0^{\pi} \\ &= -6e^{-\frac{1}{2}\pi} - 4\sin\left(\frac{1}{2}\pi\right) - \left[-6e^{-\frac{1}{2}(0)} - 4\sin\left(\frac{1}{2}(0)\right)\right] \\ &= -6e^{-\frac{\pi}{2}} - 4(1) - [-6(1) - 4(0)] \\ &= 2 - 6e^{-\frac{\pi}{2}}.\end{aligned}$$

(d) Using the sum and multiple rules,

$$\begin{aligned}\int 4\cos(-3x) - e^{-\frac{3}{2}x} \, dx &= \int 4\cos(-3x) \, dx + \int -e^{-\frac{3}{2}x} \, dx \\ &= 4\int \cos(-3x) \, dx - \int e^{-\frac{3}{2}x} \, dx \\ &= 4\left(\frac{1}{-3}\sin(-3x)\right) - \left(\frac{1}{-3/2}e^{-\frac{3}{2}x}\right) + c \\ &= -\frac{4}{3}\sin(-3x) + \frac{2}{3}e^{-\frac{3}{2}x} + c.\end{aligned}$$

(e) We will first use the sum and multiple rules to find the corresponding definite integral.

$$\begin{aligned}\int -2x^2 + e^{\cos(1)x}\,dx &= \int -2x^2\,dx + \int e^{\cos(1)x}\,dx \\ &= -2\int x^2\,dx + \int e^{\cos(1)x}\,dx \\ &= -2\left(\frac{1}{3}x^3\right) + \frac{1}{\cos(1)}e^{\cos(1)x} + c \\ &= -\frac{2}{3}x^3 + \frac{1}{\cos(1)}e^{\cos(1)x} + c.\end{aligned}$$

Hence

$$\begin{aligned}\int_{-2}^{1} -2x^2 + e^{\cos(1)x}\,dx &= \left[-\frac{2}{3}x^3 + \frac{1}{\cos(1)}e^{\cos(1)x}\right]_{-2}^{1} \\ &= -\frac{2}{3}(1^3) + \frac{1}{\cos(1)}e^{\cos(1)(1)} - \left[-\frac{2}{3}(-2)^3 + \frac{1}{\cos(1)}e^{\cos(1)(-2)}\right] \\ &= \frac{1}{\cos(1)}\left(e^{\cos(1)} - e^{-2\cos(1)}\right) - 6.\end{aligned}$$

(f) Using the sum and multiple rules,

$$\begin{aligned}&\int 2\sin(3x) - 3\sin(2x) + 2\cos(3x) - 3\cos(2x)\,dx \\ &= \int 2\sin(3x)\,dx + \int -3\sin(2x)\,dx + \int 2\cos(3x)\,dx + \int -3\cos(2x)\,dx \\ &= 2\int \sin(3x)\,dx - 3\int \sin(2x)\,dx + 2\int \cos(3x)\,dx - 3\int \cos(2x)\,dx \\ &= 2\left(-\frac{1}{3}\cos(3x)\right) - 3\left(-\frac{1}{2}\cos(2x)\right) + 2\left(\frac{1}{3}\sin(3x)\right) - 3\left(\frac{1}{2}\sin(2x)\right) + c \\ &= -\frac{2}{3}\cos(3x) + \frac{3}{2}\cos(2x) + \frac{2}{3}\sin(3x) - \frac{3}{2}\sin(2x) + c\end{aligned}$$

(g) We will first use the sum rule to find the corresponding definite integral.

$$\begin{aligned}\int e^2 + e^{2x} - 4\,dx &= \int e^2 - 4\,dx + \int e^{2x}\,dx \\ &= (e^2 - 4)x + \frac{1}{2}e^{2x} + c.\end{aligned}$$

Note that we didn't need the multiple rule here and also note that we could deal with $e^2 - 4$ all at once since $e^2 - 4$ is a constant.

Hence

$$\begin{aligned}
\int_1^3 e^2 + e^{2x} - 4\,dx &= \left[(e^2-4)x + \frac{1}{2}e^{2x}\right]_1^3 \\
&= (e^2-4)(3) + \frac{1}{2}e^{2(3)} - \left[(e^2-4)(1) + \frac{1}{2}e^{2(1)}\right] \\
&= 2(e^2-4) + \frac{1}{2}\left(e^6 - e^2\right) \\
&= \frac{1}{2}e^6 + \frac{3}{2}e^2 - 8.
\end{aligned}$$

(h) Using the sum and multiple rules,

$$\begin{aligned}
&\int -3x^{-3} + 4x^4 + 5x^{-5} + 3x^0\,dx \\
&\quad= \int -3x^{-3}\,dx + \int 4x^4\,dx + \int 5x^{-5}\,dx + \int 3x^0\,dx \\
&\quad= -3\int x^{-3}\,dx + 4\int x^4\,dx + 5\int x^{-5}\,dx + \int 3\,dx \\
&\quad= -3\left(\frac{1}{-3+1}x^{-3+1}\right) + 4\left(\frac{1}{4+1}x^{4+1}\right) + 5\left(\frac{1}{-5+1}x^{-5+1}\right) + 3x + c \\
&\quad= \frac{3}{2}x^{-2} + \frac{4}{5}x^5 - \frac{5}{4}x^{-4} + 3x + c.
\end{aligned}$$